CHAPTER IV

THE INFLUENCE OF AL-KHOWARIZMI’S ALGEBRA UPON THE
DEVELOPMENT OF MATHEMATICS

By the translators of Arabic lore we are brought from Islam to Christendom. Mathematical science in Europe was more vitally influenced by Mohammed ibn Musa than by any other writer from the time of the Greeks to Regiomontanus (1436–1476). Through his arithmetic, presenting the Hindu art of reckoning, he revolutionized the common processes of calculation and through his algebra he laid the foundation for modern analysis. Evidence of the influence of the great Arab is presented by the relatively large number of translations and adaptations of his various mathematical works which appeared before the invention of printing. Undoubtedly the earliest translation of the Arabic algebra, although not the most widely used, was that made by Robert of Chester. Probably the version published by Libri appeared shortly afterwards for, as we have mentioned, Gerard of Cremona employs the terminology of that version in algebraic work. Roger Bacon (1214–1294), too, has occasion to mention the algebra, as well as the arithmetic, and uses terms not found in Robert of Chester’s version. Bacon shows that he had but superficial familiarity with the subject, for he made incorrect statements about the fundamental elements of the algebra. Similarly, Vincent de Beauvais (about 1275) in his encyclopaedic work, Speculum Principale, refers under arithmetic to the book, qui apud Arabes mahalehe dicitur. Albertus Magnus (1193–1280) mentions the tables of Al-Khowarizmi.

Even earlier than Roger Bacon is Leonard of Pisa, whose monumental Liber abaci contains a chapter involving the title, Algebra

2 Liber xvii, Cap. V, De Arithmetica ; Liber xvii, Cap. ix, is entitled, De Computo et algorismos, and takes up representation of numbers by the Hindu numerals.
et abmuchabala. The first draft of this work was written in 1202, and in 1228 a revised and enlarged version appeared, dedicated to Michael Scotus. Woepcke has shown that Leonard drew many of his problems from Al-Khowarizmi, but some of these may have come indirectly through Abu Kamil, from whom, as I have shown, Leonard took many of his algebraic problems. In the manuscripts of the Italian’s treatise the only mention of Al-Khowarizmi is in the margin, simply Maomet, at the beginning of the section dealing with algebra; but the term algoritmus occurs for arithmetic.

In the century following Leonard of Pisa, another Italian mathematician, William of Luna, is reputed to have put Al-Khowarizmi’s algebra into the Italian language. Raffaello di Giovanni Canacci, a Florentine citizen of the fifteenth century, states in an Italian work on algebra, as yet in manuscript, that William had translated the rules of algebra out of Arabic into “our language.” Reference to his work is also made by at least three writers of the sixteenth century, the Florentine Francesco Ghaligai, the Spaniard Marco Aurel, and another Spaniard Antich Rocha of Gerona. An Italian manuscript of 1464 in the library of George A. Plimpton, Esq., of New York, does contain an Italian version of the Algebra of Al-Khowarizmi in which reference is made to William of Luna as a translator of algebra. The possibility is that we have here the version of William. The writer of the manuscript is not known, but he explicitly states that he bases his treatise on the labours of numerous predecessors in this field. One chapter, as I have shown in a recent study of this manuscript, deals with the algebra of an unknown Maestro Biagio (died 1340) and to Leonard of Pisa another section is devoted. The writer purposed also to deal with the works of a “subtle Maestro Antonio,” doubtless Antonio Mazzinghi da Peretola, who wrote a treatise on algebra called il fioretto, and a “Maestro

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2 *Extrait du Fakhrī*, p. 29.
5 *Codex Palat. 567*, Biblioteca Nazionale, Florence.
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Giouanni," but either this manuscript is incomplete or the plan was not carried out. Prominent also in the discussions is an Augustinian monk, Gratia de Castellani (about 1340), famed as a theologian. The relatively large number of names of men who had evidently attained something more than local repute in the study of algebra shows the place which it had reached in instruction.

Another prominent writer of the fourteenth century, Johannes de Muris, included a discussion of algebra in the third book of his popular Quadrupartitum numerorum. Of this section of the work of John of Meurs I have recently made a study, showing that he drew extensively from Leonard of Pisa and from Al-Khowarizmi, thus continuing the Arabic influence. Regiomontanus included the work in a list of important early works on mathematics, and further Regiomontanus refers to algebra as the ars rei et census. This corresponds to a line of the Quadrupartitum:

"Quo tamen ars minor est quam sit de censibus et rei."

Later the expression, Arte magiore, or Ars mayor, or Ars magna, was used for algebra, tracing back to this passage here given, in which ars minor refers to arithmetic as opposed to algebra. Adam Riese presents the problem, \( x^2 + 21 = 10 x \), as being found in the eleventh chapter of the third book of the Quadrupartitum; we have mentioned that this problem is one of the type problems found in Al-Khowarizmi's algebra. Another French author who gives an adaptation in Latin of the Arabic algebra is Rollandus, Canon of St. Chapelle. At the command of John, Duke of Lancaster, Rollandus wrote in the year 1424 a compendium of mathematics; the labor of composition was considerably lightened by making large extracts from the Quadrupartitum, including most of the arithmetic and algebra. A summary of the contents of the

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3 Pacioli, Summa d'aritmetica (Venice, 1494).
4 In Aurel's Libro primero, de aritmetica algebraica (Valencia, 1552).
5 Cardan, Ars Magna (Nuremberg, 1545).
manuscript is given in the *Rara Arithmetica*, but the somewhat extensive treatment of algebra is not mentioned.

The first work in the German language on algebra was an excerpt from Al-Khowarizimi which begins: "Mohammed in the book of algebra and almucabala has spoken these words ‘census, radix (root), and number.’" This is followed by two problems from the text. The manuscript which contains this brief discussion, of date 1461, is now in Munich, having been moved from the Benedictine Abbey of St. Emmeran. The first treatment in the English language appears to be that by Robert Recorde, *The Whetstone of Witte*, which was published in 1557. This work which, as I have elsewhere shown, does not display any marked originality on the part of Recorde, introduced our present symbol of equality, =, and contributed to the study of algebra in England by presenting the material in the mother tongue.

Regiomontanus seems to have been familiar with Al-Khowarizimi's work, for he not only refers to the art of thing and square (*ars rei et censu*), but also uses certain technical expressions, *restaurare defectus*, for example, similar to those in the algebra. A manuscript copy of Mohammed ibn Musa's algebra in Mr. Plimpton's collection shows astonishing similarity to the handwriting and abbreviations of Regiomontanus as well as to the form of equation used by the great German. Furthermore, some of the problems given in this manuscript, which are not part of Al-Khowarizimi's text, are discussed by Regiomontanus in his correspondence with Cardinal Blanchinus. We must suppose him to have been familiar with this text if not actually, as we suspect, the transcriber of this copy. Regiomontanus was twenty years of age when this manuscript was written (1456), and we know that he did transcribe numerous mathematical and astronomical works of historical importance. Algebra was em-

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1 *Rara*, 446–447.
ployed by Regiomontanus in his trigonometry\textsuperscript{1} in the solution of problems.

Regiomontanus has been cited by Nesselmann\textsuperscript{2} as an illustration of one who employed rhetorical algebra as opposed to syncopated or symbolical. Later writers have followed Nesselmann in the assertion that Regiomontanus used rhetorical algebra, but, whereas the statement in Nesselmann is correct in so far as the illustration which he gives is concerned, the assumption that this was the general practice of the great Teuton is an error. In fact, his correspondence with Cardinal Blanchinus shows that he had a form of equation little inferior to ours. The + sign which he uses is a ligature for \textit{et}, the minus sign a ligature for \textit{minus}, and for an equality sign he uses a single straight line. Further he has separate symbols for the various powers of the unknown up to the cube, so that Regiomontanus approached modern forms more closely than most mathematicians even of the sixteenth century. This attempted division of the history of algebra into rhetorical, syncopated, and symbolic periods is an excellent illustration of a plausible and taking theory, in historical matters, which lacks only the first essential for such a theory; namely, historical evidence. Development in mathematics, as in art and literature, does not proceed in a logical manner, but rather in waves advancing and receding, and yet withal constantly advancing.

We have mentioned the Hebrew translation of the algebra of Abu Kamil, which was made by Mordechai Finzi (about 1475 A.D.) of Mantua. Another treatise on algebra, in Hebrew, dedicated to Finzi, was written by Simon Motot.\textsuperscript{3} As the words \textit{cosa} and \textit{censo} are mentioned by Motot as being found in the works of Christian authors with which he was familiar the Italian source of his information is established, although the particular writers in question are not known. As we have above indicated, the Italians were sufficiently active in this science, so that many Italian works on algebra were available, in manuscript, at this time.

In the summer of 1486 Johann Widmann of Eger is known to

\begin{footnotesize}
\textsuperscript{1} Regiomontanus, \textit{De triangulis lietri quinque} (Nuremberg, 1533), problem 12, p. 51.
\textsuperscript{2} Nesselmann, \textit{Die Algebra der Griechen} (Berlin, 1842), p. 303.
\textsuperscript{3} G. Sacerdote, \textit{Le livre de l'algèbre et le problème des asymptotes de Simon Motot}, \textit{Revue des études juives}, 1893-1894.
\end{footnotesize}
have lectured on algebra in the university at Leipzig, and the fee for the course was set extraordinarily high, being two florins.\footnote{Wappler, *Zur Geschichte der deutschen Algebra in 15. Jahrhundert*, in *Programm, Zwickau*, 1887, pp. 9–10; also in *Zeitschrift f. Math. und Physik, Hist.-lit. Abteil.,* Vol. 45, 1900.} Widmann was in possession of the Dresden manuscript,\footnote{Codex Dresden C. 8o.} which contains Robert of Chester's version of the algebra of Al-Khowarizmi, and himself added certain algebraic problems to another part of the same manuscript dealing with algebra. The Arab Al-Kalasadi,\footnote{Woepcke, *Atti dell’ accad. pont. de’ nuovi Lincei*, Vol. XII (Rome, 1859), 230–275, 399–438.} contemporary with Widmann, wrote also on similar topics, and although he does not cite Al-Khowarizmi, yet he continues the old order of the six types of quadratic equations.

Adam Riese wrote in 1524 a work on algebra entitled, *Die Coss*, which contains, as we have noted, the problem

\[ x^2 + 21 = 10 \times. \]

Riese refers to “that most celebrated Arabic master Algebra, learned in number, whose like in computation there never was, and hardly will any one exceed him.” He refers also to the book, “named gebra and almunabola,” by this mythical Algebra. A reference to Algum is also doubtless to Al-Khowarizmi. Several contemporaries, students of the Coss, are mentioned\footnote{Berlet, *loc. cit.,* pp. 33, 34, 36, and 62 for references.} by Adam Riese, and included among these is Grammateus,\footnote{Krechenbhlin, including *Edlichen Regeln Coss*, written 1518 and published 1521.} also known as Schreiber or Scritor, to whom is credited the first algebra in print in the German language. Interesting is Riese’s note that Hans Conrad, to whom he frequently refers, paid the mathematician Andreas Alexander one florin in gold, to be taught how to solve certain types of problems by the Coss. This title is from the Italian *cosa* (Latin *res*, Arabic *sha\textsuperscript{a}i*) and connects with the use by Al-Khowarizmi, and subsequent Arabs, of the word *sha\textsuperscript{i}*, meaning *thing*, for the first power of the unknown. For centuries the title continued in circulation in Germany, and even in English appeared in the form, “the arte of cosslike numbers.”\footnote{Recorde, *loc. cit.*} Luca Paciulolo, otherwise Luca de Burgo San Sepulchro, to whom is credited the first printed work on algebra, was evidently
influenced by Al-Khowarizmi. Paciulo gives \(^1\) the equation

\[ x^2 + 10x = 39 \]

and presents the geometrical explanation as given by the Arab. In certain others of the early printed algebras the fundamental or type equations as given by Al-Khowarizmi do not appear. Of such are the works by Grammateus appearing in 1518, by Christian Rudolph in 1525,\(^2\) and by Estienne de la Roche in 1520,\(^3\) whose work is known to have been a plagiarism of the *Triparty* by Nicolas Chuquet (1484).\(^4\) However, many other writers did continue the type problems of the first systematic treatise. Thus Elia Misrachi \(^5\) in an arithmetic which appeared in Constantinople in 1534, eight years after the author's death, devotes a section to algebra, and this is to a large extent an adaptation of the *Algebra and Almucabala*. In the work by Perez de Moya (1562)\(^6\) and in the arithmetic of 1539 by Cardan we come upon the type equations. Mennher de Kempten, a Dutch mathematician, states that Algorithm was the first writer on algebra. Ghaligai, the Italian, and the Spaniard Pedro Nuñez follow the peculiar order of equations found in Al-Khowarizmi. In these and other ways we might trace through the centuries the persistent influence, direct and indirect, of our Arabic author, but that is beside our present purpose.

Among the writers who made a serious study of Robert of Chester's translation we must place Johann Scheybl (1494–1570), who was professor of mathematics at Tübingen from about 1550 to the time of his death. He was the author of an algebra which appeared in two editions in Paris, 1551 and 1552. This treatment of algebra was first published in 1550 by Scheybl, prefixed to his Greek and Latin edition of the first six books of Euclid. Scheybl

\(^1\) *Summa de arithmetica* (Venice, 1494), fol. 146 rect.
\(^2\) I have not seen a copy of this edition. My remark is based upon *Die Coss Christoff's Rudolff's* (Königsberg, 1553) by Stifel. From certain notes about the history of the terminology and the words *dragma, res* and *substantia*, and the like, it appears that Stifel had seen a copy of Robert of Chester's version.
\(^3\) Smith, *Rara Arithmetica*, p. 128.
\(^6\) *Arithmetica practica, y speculativa* (Salamanca, 1562).
prepared for publication the Latin version of Al-Khowarizmi’s algebra as translated by Robert of Chester. His manuscript copy is now in the Columbia University library. The title page reads, in translation: ‘A brief and clear exposition of the rules of algebra by Johann Schebyl, Professor of Euclid in the famous University of Tübingen. To this is added the work, *On given numbers*, by that most excellent mathematician Jordanus. Furthermore there is presented the book containing the demonstrations of the rules of algebraic equations, written some time ago in Arabic. All of these are now published for the first time by the above-mentioned Schebyl. These are corrected as far as possible and illustrated by appropriate and useful examples.’

The algebra contained in this manuscript is not the same as the published work mentioned above. However, the method of treatment is not materially different. The work by Jordanus Nemorarius, entitled *De numeris datis*, dates from the early part of the thirteenth century. The importance of the work, chiefly with respect to the development of algebra, is well attested by the fact that Regiomontanus and Maurolycus both planned to publish the work, although neither carried the plan to completion. The work was published in 1879 by Treutlein. The Schebyl version contains the complete list of 113 propositions to which Chasles made reference in 1841. These are divided into four books containing respectively 30, 26, 22 and 35 propositions. Schebyl adds solutions by the rules of algebra in which he employs the same algebraic notation as in his published algebra, but he does not give the complete text of the work of Jordanus. The other two works in the manuscript are presented in this monograph. The manuscript was carefully prepared, but, for some reason which we do not know, the publication was not accomplished.

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1 Brevis ac dilucida regularum Algebræ descripțio, autore Joanne Scheubello, in indicata Tubingena academia Euclidis professore ordinario. Huic accedet liber consumatissimi mathematici Jordonii, de datis. Liber praeterea, continens demonstrationes aequationum regularum Algebræ, Arabice olim conscriptus.

Quae (corr. Qui) ambo ab eodem Scheubello nunc primum, quantum fieri potuit, emendato (corr. emendavit) in lucem aedita, et aptissimis atque utilibus exemplis illustrata sunt.


4 Frequently adding, *Segueitur solutio ex regula Algebræ*. 
Plate II

Codex Dresdensis C. 80, Fol. 340b
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A few words about the life of Scheybl 1 may be of interest. His student days include an early stay at the University of Vienna, made famous in mathematical studies by Peurbach and Regiomontanus. In 1532 Scheybl matriculated at Tübingen, which was a stronghold of Protestantism, and in 1535 he was a student there. In 1540 he was Magister in Tübingen, and four years later Docent in mathematics. After another period of about five years we find him Professor (ordinarius) of Euclid, and in 1555 Professor of Euclid and Arithmetic. How little some aspects of university life have changed during four centuries is shown by the fact that Scheybl twice, in 1551 and 1562, requested of the university authorities an increase of salary in order that he might pay his debts and obtain the necessaries of life. In addition to the treatises on geometry and algebra which have been mentioned, Scheybl published other works on geometry and arithmetic.

As late as the end of the sixteenth century an able mathematician, Adrien Romain, deemed the algebra of Al-Khowarizmi sufficiently worthy of serious study to justify him in publishing a commentary on the work. The version upon which Romain based his study is that of Robert of Chester. In the course of his commentary he gives small portions of this translation. At the time of writing this, according to Bosmans 2 in 1598 or 1599, Romain was teaching in Würzburg. His student and teaching life covered periods of residence in Germany, Italy, and Poland, besides his native Louvain where he studied and taught.