CHAPTER VI

Preface and Additions Found in the Arabic Text of Al-Khowarizmi's Algebra

The Arabic text of Al-Khowarizmi's algebra published by Rosen contains an author's preface which is not found either in the translation published by Libri, or in that by Robert of Chester. As this reveals his conception of the purpose of the algebra, as well as some of the causes which led him to undertake the work, we present it here in the translation by Rosen. Such prefaces in Arabic works usually, just as this one, contained invocations to the Deity and to Mohammed his prophet: in consequence the Christian translators, who were commonly connected with the Church, were wont to leave them out. A summary of the sections in the Arabic text which appear in neither of the Latin translations is also given since the Arabic-English work by Rosen is not widely available, and since these additions show that Al-Khowarizmi had grasped the possibility of the application of the algebra to geometry and trigonometry. This application is frequently neglected to-day by teachers of elementary algebra.

The Author's Preface

"In the Name of God, gracious and merciful!"

"This work was written by Mohammed ben Musa, of Khowarezm. He commences it thus:

"Praised be God for his bounty towards those who deserve it by their virtuous acts: in performing which, as by him prescribed to his adoring creatures, we express our thanks, and render ourselves worthy of the continuance (of his mercy), and preserve ourselves from change: acknowledging his might, bending before his power, and revering his greatness! He sent Mohammed (on whom may the blessing of God repose!) with the mission of a prophet, long after any messenger from above had appeared, when justice had fallen into neglect, and when the true way of life was sought for in vain. Through him he cured of blindness, and saved through him from perdition, and increased through him what before was small, and collected through him what before was scattered. Praised be God our Lord! and may his glory increase, and may all his names be hallowed — besides whom there is no God; and may his benediction rest on Mohammed the prophet and on his descendants!"

1 Rosen, The Algebra of Mohammed ben Musa, pp. 1-4.

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"The learned in times which have passed away, and among nations which have ceased to exist, were constantly employed in writing books on the several departments of science and on the various branches of knowledge, bearing in mind those that were to come after them, and hoping for a reward proportionate to their ability, and trusting that their endeavors would meet with acknowledgement, attention, and remembrance—content as they were even with a small degree of praise; small, if compared with the pains which they had undergone, and the difficulties which they had encountered in revealing the secrets and obscurities of science.

"Some applied themselves to obtain information which was not known before them, and left it to posterity; others commented upon the difficulties in the works left by their predecessors, and defined the best method (of study), or rendered the access (to science) easier or placed it more within reach; others again discovered mistakes in preceding works, and arranged that which was confused, or adjusted what was irregular, and corrected the faults of their fellow-laborers, without arrogance towards them, or taking pride in what they did themselves.

"That fondness for science, by which God has distinguished the Imam al Mamun, the Commander of the Faithful (besides the caliphat which He has vouchsafed unto him by lawful succession, in the robe of which He has invested him, and with the honours of which He has adorned him), that affability and condescension which he shows to the learned, that promptitude with which he protects and supports them in the elucidation of obscurities and in the removal of difficulties,—has encouraged1 me to compose a short work on Calculating by (the rules of) Completion and Reduction, confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, law-suits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computation, and other objects of various sorts and kinds are concerned—relying on the goodness of my intention therein, and hoping that the learned will reward it, by obtaining (for me) through their prayers the excellence of the Divine mercy: in requital of which, may the choicest blessings and the abundant bounty of God be theirs! My confidence rests with God, in this as in everything, and in Him I put my trust. He is the Lord of the Sublime Throne. May His blessing descend upon all the prophets and heavenly messengers!"

The Arabic version differs from the Latin translations which have come down to us, in giving an extended discussion of inheritance problems and also in discussing geometrical measurements. In the English translation by Rosen the inheritance problems, involving largely legal questions rather than algebraical ones, occupy 79 pages as opposed to 70 for the algebra proper. The mensuration problems take some sixteen pages of text. The formulas are

1 Several writers have asserted that the work of Al-Khowarizmi was written at the request of the caliph. The text shows that this is not Al-Khowarizmi's statement of the case.

given for the area of a square and triangle. Three formulas are
given for the circumference of a circle and the writer evidently
recognizes them all as approximations. The formulas are:

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\begin{align*}
(1) & \quad \epsilon = \frac{3}{10} d, \\
(2) & \quad \epsilon = \sqrt{10} d^2 \\
(3) & \quad \epsilon = \frac{62832}{20000} d
\end{align*}
\]

The area of a circle is given as \( A = d^2 - \frac{1}{2} d^2 \) of \( \frac{1}{3} d^2 \). Other
simple areas and volumes are discussed. Application of the
algebra is found in two problems. One of these deals with finding
the altitude of a triangle of which the sides are given; the other
with inscribing a square in a given triangle.

As the problems, on finding the altitude of a triangle, being
given the lengths of the sides, and on inscribing in an isosceles
triangle a square, show that Al-Khowarizmi had an appreciation
of the possibilities of the algebra, I present one of the problems,
following Rosen's translation.

"If some one says: 'There is a triangular piece of land, two of its sides having
10 yards each, and the basis 12; what must be the length of one side of a quadrate
situated within such a triangle?' the solution is this. At first you ascertain the
height of the triangle, by multiplying the moiety of the basis, (which is six) by itself,
and subtracting the product, which is thirty-six, from one of the two short sides
multiplied by itself, which is one-hundred; the remainder is sixty-four; take the
root from this; it is eight. This is the height of the triangle. Its area is, therefore,
fourty-eight yards: such being the product of the height multiplied by the moiety of
the basis, which is six. Now we assume that one side of the quadrate inquired for
is thing. We multiply it by itself; thus it becomes a square, which we keep in mind.
We know that there must remain two triangles on the two sides of the quadrate, and
one above it. The two triangles on both sides of it are equal to each other: both
having the same height and being rectangular. You find their area by multiplying
thing by six less half a thing, which gives six things less half a square. This is the
area of both the triangles on the two sides of the quadrate together. The area of
the upper triangle will be found by multiplying eight less thing, which is the height, by half
one thing. The product is four things less half a square. This altogether is equal to
the area of the quadrate plus that of the three triangles; or, ten things are equal to forty-
eight, which is the area of the great triangle.
One thing from this is four yards and four-
fifths of a yard; and this is the length of any
side of the quadrate. Here is the figure:
The inheritance problems occupy a large part of the original work; the inclusion of one of these problems here will perhaps not be amiss. Only the first of the problems is given since the following problems are of the same general nature, involving other legal peculiarities.

"A man dies, leaving two sons behind him, and bequeathing one-third of his capital to a stranger. He leaves ten dirhems of property and a claim of ten dirhems upon one of the sons.

"Computation: You call the sum which is taken out of the debt thing. Add this to the capital which is ten dirhems. The sum is ten and thing. Subtract one-third of this, since he has bequeathed one-third of his property, that is, three dirhems and one-third of thing. The remainder is six dirhems (and two-thirds) and two-thirds of thing. Divide this between the two sons. The portion of each of them is three dirhems and one-third plus one-third of thing. This is equal to the thing which was sought for. Reduce it, by removing one-third from thing, on account of the other third of thing. There remain two-thirds of thing, equal to three dirhems and one-third. It is then only required that you complete the thing, by adding to it as much as one-half of the same; accordingly, you add to three and one-third as much as one-half of them: This gives five dirhems, which is the thing that is taken out of the debts."

The legal point involved in the problem given is that a son who owes to the estate of his father an amount greater than the son's portion of the estate, retains, in any event, the whole sum which he owes. Part is regarded as his share of the estate, and the remainder as a gift from the father. The above problem would have given exactly the same numerical results for any debt from five dirhems up; however, if there were a claim of four dirhems against one of the sons, instead of ten, the debtor son would have received in cash $\frac{2}{3}$ of one dirhem, the other son four and $\frac{2}{3}$ dirhems, and the stranger four and $\frac{3}{3}$ dirhems.

In algebraical symbolism, the equation is $\frac{2}{3}(10 + x) = 2x$ whence $x = 5; 10 + x$ is the total estate left, and $x$ is the share of each son.
Plate III.